Math 324 Quiz 2

February 23, 2017

Problem 1. Convert the point $(3, 4, 5)$ from cartesian coordinates to spherical coordinates.

Solution 1. Remember, the order for spherical is (ρ, θ, ϕ)! Then

$$\rho = \sqrt{x^2 + y^2 + z^2} = \sqrt{3^2 + 4^2 + 5^2} = \sqrt{50} = 5\sqrt{2}$$

$$\theta = \tan^{-1}(y/x) = \tan^{-1}(4/3).$$

$$\phi = \cos^{-1}(z/\sqrt{x^2 + y^2 + z^2}) = \cos^{-1}(5/(5\sqrt{2})) = \cos^{-1}(1/\sqrt{2}) = \pi/4.$$

Therefore the point is $(5\sqrt{2}, \tan^{-1}(4/3), \pi/4)$.

Problem 2. Convert the point $(1, \pi/6, \pi/2)$ from spherical coordinates to cartesian coordinates.

Solution 2. Remember, the order for spherical is (ρ, θ, ϕ)! Then

$$x = \rho \sin\phi \cos\theta = 1 \sin(\pi/2) \cos(\pi/6) = \sqrt{3}/2.$$

$$y = \rho \sin\phi \cos\theta = 1 \sin(\pi/2) \sin(\pi/6) = 1/2.$$

$$z = \rho \cos\phi = 1 \cos(\pi/2) = 0.$$

Problem 3. Determine the Jacobian of the linear transformation

$$u = x + 2y$$
$$v = 2y$$

Solution 3. We have that

$$x = u - v$$
$$y = v/2$$

and therefore the Jacobian is

$$J = \det \begin{pmatrix} 1 & -1 \\ 0 & 1/2 \end{pmatrix} = 1/2.$$

Problem 4. Using spherical coordinates, set up an integral to find the integral of $f(x, y, z) = x + y$ over the region R above the cone $z = 2\sqrt{x^2 + y^2}$ and below the sphere $x^2 + y^2 + z^2 = 16$. **Do not evaluate it**

Solution 4. The cone $z = 2\sqrt{x^2 + y^2}$ in spherical is given by

$$\rho \cos(\phi) = 2\rho \sin(\phi).$$

This simplifies to $\tan(\phi) = 1/2$, and therefore $\phi = \tan^{-1}(1/2)$. Since we're looking at the region above the cone, the range of ϕ values should therefore be 0 to $\tan^{-1}(1/2)$. The range of θ values is $0 \leq \theta \leq 2\pi$, and the range of ρ values is $0 \leq \rho < 4$ (because we're inside a sphere of radius 4). Therefore the integral is

$$\int_0^{2\pi} \int_0^{\tan^{-1}(1/2)} \int_0^4 (\rho \sin\phi \cos\theta + \rho \sin\phi \sin\theta)\rho^2 \sin\phi d\rho d\phi d\theta.$$