Math 324 Quiz 3 Practice

February 15, 2017

Problem 1. Calculate the directional derivative of

$$f(x, y, z) = x^2y + y^2z + z^2x$$

in the direction of $\langle 1, 0, -1\rangle$ at the point $(1, 2, 3)$.

Solution 1. We first calculate the gradient to be

$$\nabla f = \langle 2xy + z^2, x^2 + 2yz, y^2 + 2zx\rangle,$$

which at the point $(1, 2, 3)$ takes the value $\langle 13, 13, 10\rangle$. The direction we want to take the derivative in is (normalized) $\vec{u}\langle 1/\sqrt{2}, 0, -1/\sqrt{2}\rangle$, so therefore

$$D_{\vec{u}}f = \langle 13, 13, 10\rangle \cdot \langle 1/\sqrt{2}, 0, -1/\sqrt{2}\rangle = 3/\sqrt{2}.$$

Problem 2. Let $f(x, y, z), g(x, y, z)$ be functions. Show that taking the gradient distributes over addition, ie.

$$\nabla(f + g) = \nabla f + \nabla g.$$

Solution 2. We calculate

$$\begin{aligned}\nabla(f + g) &= \langle (f + g)_x, (f + g)_y, (f + g)_z\rangle \\ &= \langle f_x + g_x, f_y + g_y, f_z + g_z\rangle \\ &= \langle f_x, f_y, f_z\rangle + \langle g_x, g_y, g_z\rangle \\ &= \nabla f + \nabla g.\end{aligned}$$

Problem 3. Calculate the gradient of

$$f(x, y) = \tan^{-1}(y/x).$$

Solution 3. We have

$$f_x = \frac{1}{1 + (y/x)^2}(-y/x^2) = \frac{-y}{x^2 + y^2},$$

and also

$$f_y = \frac{1}{1 + (y/x)^2}(1/x) = \frac{x}{x^2 + y^2},$$

and therefore

$$\nabla f = \langle -y/(x^2 + y^2), x/(x^2 + y^2)\rangle.$$

Problem 4. Show that

$$\vec{F}(x, y) = \langle -y/(x^2 + y^2), x/(x^2 + y^2) \rangle$$

is a conservative vector field.

Solution 4. Since $\vec{F}(x, y) = \nabla f$ for $f(x, y)$ the function from the previous problem, the field $\vec{F}(x, y)$ is conservative.