

Math 324 Quiz 5

March 7, 2017

Problem 1. For each of the following statements, write TRUE if the statement is true and FALSE if the statement is false (in big capitalized letters)! It is not necessary to show your work, and you may assume all vector fields and scalar functions are smooth. Note: not all expressions make sense.

- (a) for any vector field \vec{F} , $\operatorname{div}(\operatorname{curl}(\vec{F})) = 0$
- (b) for any vector field \vec{F} , $\operatorname{curl}(\operatorname{div}(\vec{F})) = \langle 0, 0, 0 \rangle$
- (c) for any scalar function f , $\operatorname{curl}(\operatorname{grad}(f)) = \langle 0, 0, 0 \rangle$
- (d) for any scalar function f , $\operatorname{div}(\operatorname{grad}(f)) = 0$
- (e) if \vec{F} is divergence-free and non-rotational, then $\vec{F} = \nabla f$ for f a solution of Laplace's equation $\Delta f = 0$.

Problem 2. For each of the following vector fields \vec{F} , determine whether the vector field is conservative. If it is, **bonus pts** for finding f with $\nabla f = \vec{F}$.

(a)

$$\vec{F}(x, y, z) = \langle 1 + y^2 z^3, 2y + 2xyz^3, \cos(z) + 3xy^2 z^2 \rangle.$$

(b)

$$\vec{F}(x, y, z) = \langle yz \cos(xy), xz \cos(xy), xy \sin(xy) \rangle.$$