Math 324 Quiz 5 Practice

February 28, 2017

Problem 1. Suppose that $f(x, y, z)$ is a scalar function and that $\vec{F}(x, y, z) = \langle F_1(x, y, z), F_2(x, y, z), F_3(x, y, z)\rangle$ is a three-dimensional vector field. Show that

$$\nabla \cdot (f\vec{F}) = f\nabla \cdot \vec{F} + \vec{F} \cdot \nabla f.$$

Problem 2. Suppose that $\vec{F}(x, y, z)$ is a three-dimensional vector field. Show that

$$\nabla \times (\nabla \times \vec{F}) = \nabla(\nabla \cdot \vec{F}) - \Delta\vec{F},$$

where here

$$\Delta\vec{F} = \langle \Delta F_1, \Delta F_2, \Delta F_3 \rangle\,,$$

for $\Delta = \partial^2/\partial x^2 + \partial^2/\partial_y^2 + \partial^2/\partial_z^2$ the Laplacian operator.

Problem 3. For each value of $\vec{F}$, determine whether or not $\vec{F}$ is conservative. If it is, find a function f such that $\vec{F} = \nabla f$.

(a) $\vec{F}(x, y, z) = \langle 3xy^2z^2, 2x^2yz^3, 3x^2y^2z^2 \rangle$

(b) $\vec{F}(x, y, z) = \langle e^{yz}, xze^{yz}, xye^{yz} \rangle$